JOURNAL OF APPLIED ECONOMICS AND MANAGEMENT OF ORGANIZATIONS (JAEMO)

REVUE D'ECONOMIE APPLIQUÉE ET DE GESTION DES ORGANISATIONS (REAGO)

ISSN 1987-1694

Vol. 2 – N° 1 – december / décembre 2025

Email : jaemoeditor@gmail.com

Online : www.lread.ml

http://esjindex.org/search.php?id=7919

https://www.ascidatabase.com/masterjournallist.php?v=17501

EDITORIAL LINE

Journal of Applied Economics and Management of Organizations (JAEMO) publishes scientific manuscripts in the field of economics and management sciences as well as in very close fields. Nowadays, economics and management sciences have progressed a lot to the point that they have made alliances with neighboring sciences. This journal aims to promote very specialized, original and innovative scientific research.

Currently, the journal publishes in two languages: French and English. The JAEMO journal is biannual (one issue in June and the second in December). The articles to be published by the journal must deal with a very clear problem in the fields as defined above, present the conceptual framework, the well-provided methodology, the quality data with rigorous treatment and the results that can contribute to scientific research and applied policies.

Theoretical articles are also welcome as long as they indicate the modeled stylized facts and the intuitions on advanced theories. These types of manuscripts must demonstrate advanced conceptualization and use rigorous methodological tools to pave the way for future empirical verifications. In any case, the journal remains very demanding on scientific rigor.

PUBLICATION AND FORMAT

The journal is published in electronic and paper versions under the ISSN 1987-1694 twice a year. The online version is available in PDF format in the form of the complete volume or offprints. The edited version is in A4 format.

LIGNE EDITORIALE DE LA REVUE

Journal of Applied Economics and Management of Organizations (JAEMO) publie les manuscrits scientifiques dans le domaine des sciences économiques et de gestion ainsi que dans les domaines très proches. De nos jours, les sciences économiques et de gestion ont beaucoup progressé au point qu'elles ont fait des alliances avec les sciences voisines. Cette revue se veut faire la promotion de la recherche scientifique très pointue, originale et innovante.

Actuellement, la revue publie dans deux langues qui sont le français et l'anglais. La revue JAEMO est semestrielle (un numéro le mois de juin et le second en décembre). Les articles à publier par la revue doivent porter sur une problématique très claire dans les domaines tels que définis ci-dessus, présenter le cadre conceptuel, la méthodologie bien fournie, les données de qualité avec un traitement rigoureux et les résultats pouvant contribuer à la recherche scientifique et aux politiques appliquées.

Les articles théoriques sont aussi les bienvenus pour autant qu'ils indiquent les faits stylisés modélisés et les intuitions sur des théories avancées. Ces types de manuscrits doivent faire preuve d'une conceptualisation avancée et user d'outils méthodologiques rigoureux pour ouvrir la voie à des vérifications empiriques futures. En tout état de cause, la revue reste très exigeante sur la rigueur scientifique.

PARUTION ET FORMAT

La revue parait en version électronique et en version papier sous l'ISSN ISSN 1987-1694 deux fois par an. La version en ligne est disponible en format PDF sous forme du volume complet ou des tirés à part. La version éditée est en format A4.

SCIENTIFIC COUNCIL MANAGEMENT SCIENCES

CONSEIL SCIENTIFIQUE SCIENCES DE GESTION

MANAGEMENT SCIENCES READING COMMITTEE

COMITE DE LECTURE SCIENCES DE GESTION

SUMMARY / SOMMAIRE

Journal of Applied Economics and Management of Organizations (JAEMO)
Revue d'économie appliquée et de gestion des organisations
ISSN 1987-1694 - Vol. 2 – N°1

Asymptotic tests of exponential variation index against alternatives

Tests asymptotiques de l'indice de variation exponentielle contre des alternatives

Aboubacar Yacouba TOURE[1] – Amadou BAMBA[2] – Alhousseyni Amadou MAÏGA[3*]

1- Université des Sciences Sociales et de Gestion de Bamako, Mali ; toureaboubacaryacoubaaliou@gmail.com
2- Université des Sciences Sociales et de Gestion de Bamako ; Laboratoire de Recherche en Economie Appliquée au Développement ; abambisco@gmail.com
3 - (Corresponding author) ; Université des Sciences Sociales et de Gestion de Bamako ; Ministère de l'Economie et des Finances ;* maigis@yahoo.fr

Abstract

Variation index with respect to exponential is a classical tool that are widely used in Statistic. This paper aims to gather and study the empirical probabilities of type II error with respect to undervariation and overvariation alternatives. We establish some hypotheses and critical regions and propose numerical studies based on these critical regions. Some concluding remarks are made, including challenging problems.

Keywords: Continuous distribution, equivariation, overvariation, probability of type I error, probability of type II error, undervariation.

Classification JEL : C15

Résumé **:**
L'indice de variation par rapport à la loi exponentielle est un outil classique largement utilisé en statistique. Cet article vise à rassembler et à étudier les probabilités empiriques d'erreur de type II face aux alternatives de sous-variation et de sur-variation. Nous établissons plusieurs hypothèses ainsi que des régions critiques et proposons des études numériques fondées sur ces régions critiques. Quelques remarques conclusives sont formulées, notamment en mettant en évidence des problèmes ouverts et des pistes de recherche.

Mots-clés : *Distribution continue, équi-variation, sur-variation, probabilité d'erreur de type I, probabilité d'erreur de type II, sous-variation.*

Classification JEL : *C15*

1. Introduction

In statistical modelling which is an important step of data analysis in many fields of scientific research, one needs to specify a probability distribution that accounts as accurately as possible the variability observed in the data. In practice it is often considered that certain phenomena such as over/undervariation or zero mass for continuous data (e.g., Abid et al. 2020).

Abid et al. (2020) have introduced the exponential variation index (VI), so-called Jørgensen variation index, for continuous random variable X on $(0,\infty)$ as $VI(X) = Var\ X/(\mathbb{E}X)^2$. It can be viewed as the squared coefficient of variation and, also, it is used in the framework of reliability to discriminate distribution of increasing/decreasing failure rate on the average (IFRA/DFRA) ; see, e.g., Barlow and Proschan (1981) in the sense of the coefficient of variation. A multivariate extension has been well defined and discussed in literature (Kokonendji et al. 2020). There are many interpretable mechanisms leading to this phenomenon which makes it possible to classify count distributions and makes inference; see, e.g., Touré et al. (2020,2022) for approximative statistical tests. It is not easy to construct an exact statistical test for the index VI based on its plug-in estimator in order to assess the amount of the variability ; see, e.g., Lam (1980) ; Touré et al. (2020,2022).

Furthermore Touré et al. (2022) have introduced the following test statistic :

$$T_{n,V} = \sqrt{\frac{n}{2 + V''(\bar{X}_n)}}\left(\frac{S_n^2}{V(\bar{X}_n)} - 1\right) \rightsquigarrow \mathcal{N}(0;1). \quad (\mathbf{1})$$

where $V''(\cdot)$ denotes the second derivative of the variance function, S_n^2 is the sample variance, $\bar{X}_n$ is the sample mean, $\rightsquigarrow$ stands for convergence in distribution and $\mathcal{N}(0;\tau^2)$ is the centered normal distribution with variance $\tau^2 = 2 + V''(\mu) > 0$.

In their work, the authors used the test statistic $T_{n,V}$ to define critical regions for two-tailed and one-tailed tests. However, they did not attempt to determine the power of these tests under alternatives when the exponential model is inadequate. To our knowledge, no study has established the power of exponential tests for alternatives to the inadequacy of the exponential model.

The two main goals of this paper are, firstly, to propose asymptotic two-tailed and one-tailed exponential tests and, then to propose undervariation and overvariation alternatives in order to study the power of these tests. To get this treatment, we derive from (1) the statistic $T_{n,E} = \sqrt{\frac{n}{4}}\left(\frac{S_n^2}{\bar{X}_n^2} - 1\right) \rightsquigarrow \mathcal{N}(0;1)$ with respect to exponential model and which we will use for the empirical probabilities of type I error in two-tiled and one-tiled tests.

The rest of the paper is structured as follows. Section 2 will be devoted to present the hypotheses to test and critical regions of the test statistics that have produced good results. In Section 3 we analyse the results of some numerical studies. Finally, Section 4 concludes with some remarks and possible extensions.

2. Hypotheses to test and critical regions

In this Section, we present the following five families of the hypotheses to test with their corresponding critical regions (1) and for which we find them interesting in practice :

(i) $H_{0,EE}: \sigma^2 = \mu^2$ against $H_{1,EE}: \sigma^2 \neq \mu^2$ for the two-tiled test with respect to the exponential equivariation and, therefore, $H_{0,EE}$ is rejected if

$$T_{n,EE} = \left|\sqrt{\frac{n}{4}}\left(\frac{S_n^2}{\bar{X}_n^2} - 1\right)\right| > u_{1-\alpha/2}; \qquad \textbf{(2)}$$

(ii) $H_{0,EU}: \sigma^2 = \mu^2$ against $H_{1,EU}: \sigma^2 < \mu^2$ for the one-tiled test with respect to the exponential undervariation and, therefore, $H_{0,EU}$ is rejected if

$$T_{n,EU} = \sqrt{\frac{n}{2}}\left(\frac{S_n^2}{\bar{X}_n^2} - 1\right) < u_\alpha; \qquad \textbf{(3)}$$

(iii) $H_{0,EO}: \sigma^2 = \mu$ against $H_{1,EO}: \sigma^2 > \mu$ for the one-tiled test with respect to the exponential overvariation and, therefore, $H_{0,EO}$ is rejected if

$$T_{n,EO} = \sqrt{\frac{n}{2}}\left(\frac{S_n^2}{\bar{X}_n^2} - 1\right) > u_{1-\alpha}; \qquad \textbf{(4)}$$

(iv) $H_{0,IG}: \sigma^2 \geq \mu^3$ against $H_{1,IG}: \sigma^2 < \mu^3$ for the undervariation with respect to inverse Gaussian distribution and, thus, $H_{0,IG}$ is rejected if

$$T_{n,IG} = \sqrt{\frac{n}{2+6\bar{X}_n}}\left(\frac{S_n^2}{\bar{X}_n^3} - 1\right) < u_\alpha; \qquad \textbf{(5)}$$

(v) $H_{0,IG}: \sigma^2 \leq \mu^3$ against $H_{1,IG}: \sigma^2 > \mu^3$ for the overvariation with respect to inverse Gaussian distribution and $H_{0,IG}$ is rejected if

$$T_{n,IG} = \sqrt{\frac{n}{2+6\bar{X}_n}}\left(\frac{S_n^2}{\bar{X}_n^3} - 1\right) > u_{1-\alpha}. \qquad \textbf{(6)}$$

These critical regions have led us to carry out simulation studies which results can be found in Tables 1-5 in the next Section.

3. Simulation studies

In this Section, we first present the empirical probabilities of type I error for the two-tailed and one-tailed exponential tests. Then, we compute the empirical probabilities of type II error for the inverse Gaussian undervariation and inverse Gaussian overvariation alternatives, respectively. All this simulations work was carried out using the $\mathcal{R}$ software ($\mathcal{R}$ Core Team, 2025).

The extreme quantiles of the distribution of a test statistic are often those of interest for the implementation of a hypothesis test based on this statistic. Good approximations are therefore needed for these extreme quantiles when the exact distribution of the test statistic is unknown.

The aim of simulations whose results are presented below is to evaluate empirically the quality of the approximation of the extreme quantiles of the test statistic $T_{n,V}$ by the nominal quantiles corresponding to the limit laws with respect to the sample sizes n. The simulation study was designed with

- sample sizes $n =$ 30, 50, 100, 500 and 1000 ;

- Poisson mean $\mu = 0.5$, 1, 5, 10 and 20 ;
- number of trials of binomial distribution $N = 25$ and 100 ;
- parameter of negative binomial distribution $\lambda = 25$ and 100 ;
- significance level $\alpha = 0.05$;
- number of replications $r = 50000$.

Note that we performed 50000 replications for each simulated sample.

3.1.Empirical probabilities of type I error

Given that the critical region of the test is determined from the asymptotic distribution of $T_{n,E}$, we expect the probabilities of type I error to deviate from the nominal value corresponding to these critical regions. A probability of type I error significantly greater than the nominal value 0.05 would indicate that the precision of the approximation of the extreme quantiles of the distribution of test statistics by those of its limiting distribution is not sufficient. The results of probabilities of type I error for the two-tailed and one-tailed tests are reported in Tables 1-3 where values above 0.05 are in bold.

3.1.1. Exponential two-tiled test

The results in Table 1 show that the empirical probabilities of type I error are lower than the nominal value 0.05 and whatever the sample size n. Therefore, one can said that the extreme quantiles of the distribution of $T_{n,E}$ are approximated with sufficient accuracy by those of the centred and reduced gaussian distribution.

Table 1 : Empirical probabilities of type I error with respect to sample size n for exponential two-tiled test (i).

n	$\mu = 0.5$	$\mu = 1$	$\mu = 5$	$\mu = 10$	$\mu = 20$
30	0.0315	0.0314	0.0316	0.0323	0.0312
50	0.0353	0.0341	0.0333	0.0344	0.0317
100	0.0395	0.0384	0.0388	0.0385	0.0389
500	0.0462	0.0461	0.0456	0.0453	0.0467
1000	0.0473	0.0485	0.0472	0.0491	0.0468

Source : Authors' calculation

3.1.2. Exponential one-tiled test

For 2-3, the empirical probabilities of type I error are lower thant 0.05 or not significantly different from this value and whatever the sample size n. It is difficult to choose between undervariation and overvariation in view of the results. We cannot discriminate between the equivariation exponential model and undervariation or overvariation.

Table 2 : Empirical probabilities of type I error with respect to sample size n for exponential one-tiled undervariation test (ii).

$\boldsymbol{n}$	$\boldsymbol{\mu = 0.5}$	$\boldsymbol{\mu = 1}$	$\boldsymbol{\mu = 5}$	$\boldsymbol{\mu = 10}$	$\boldsymbol{\mu = 20}$
30	0.0034	0.0039	0.0034	0.0031	0.0033
50	0.0106	0.0095	0.0102	0.0097	0.0101
100	0.0192	0.0193	0.0196	0.0209	0.0195
500	0.0364	0.0368	0.0355	0.0390	0.0379
1000	0.0417	0.0417	0.0405	0.0419	0.0407

Source : Authors' calculation

Table 3 : Empirical probabilities of type I error with respect to sample size n for exponential one-tiled overvariation test (iii).

$\boldsymbol{n}$	$\boldsymbol{\mu = 0.5}$	$\boldsymbol{\mu = 1}$	$\boldsymbol{\mu = 5}$	$\boldsymbol{\mu = 10}$	$\boldsymbol{\mu = 20}$
30	0.0464	0.0476	0.0457	0.0463	0.0460
50	0.0498	**0.0501**	**0.0508**	**0.0504**	**0.0503**
100	**0.0504**	0.0477	**0.0506**	0.0440	0.0465
500	0.0463	0.0448	0.0470	0.0450	0.0447
1000	0.0477	0.0462	0.0466	0.0464	**0.0509**

Source : Authors' calculation

Thus, the extreme quantiles of the distribution of $T_{n,E}$ are approximated with sufficient accuracy by those of the centred and reduced gaussian distribution.

3.2.Empirical probabilities of type II error

In this Section as an alternative to the inadequacy of the exponential model, we choose the inverse Gaussian distribution for both undervariation and overvariation. The empirical probabilities of type II error are given in Tables 4 and 5.

3.2.1 Inverse Gaussian as under/over-variation alternative

The inverse Gaussian distribution can be used for both undervariation and overvariation alternatives when the results observed in the data do not allow the exponential distribution to be validated. The inverse Gaussian distribution is defined as follow :

$$f(x;\mu,\lambda) = \left(\frac{\lambda}{2\pi x^3}\right)^{1/2} \exp\left\{\frac{-(x-\mu)^2\lambda}{2\mu^2 x}\right\} \mathbf{1}_{(0,\infty)}(x); \quad \textbf{(7)}$$

with $\mu > 0, \lambda > 0$. The mean and variance are μ and μ^3/λ, respectively.

With regard to Tables 4-5, the empirical probabilities of type II error are well above 0.05 and whatever the sample size n.

Table 4 : Empirical probabilities of type II error with respect to sample size n for inverse Gaussian undervariation alternative test (iv) when $\lambda = 3$

n	$\mu = 0.5$	$\mu = 1$	$\mu = 5$	$\mu = 10$	$\mu = 20$
30	0.9989	0.9993	0.9995	0.9997	0.9999
50	0.9943	0.9975	0.9980	0.9988	0.9997
100	0.9858	0.9879	0.9896	0.9923	0.9939
500	0.9867	0.9871	0.9887	0.9913	0.9935
1000	0.9597	0.9668	0.9717	0.9899	0.9918

Source : Authors' calculation

Table 5 : Empirical probabilities of type II error with respect to sample size n for inverse Gaussian overvariation alternative test (v) when $\lambda = 1$

n	$\mu = 0.5$	$\mu = 1$	$\mu = 5$	$\mu = 10$	$\mu = 20$
30	0.9597	0.9719	0.9968	0.9995	0.9999
50	0.9535	0.9632	0.9910	0.9977	0.9996
100	0.9496	0.9555	0.9798	0.9916	0.9981
500	0.9437	0.9465	0.9567	0.9662	0.9771
1000	0.9431	0.9419	0.9521	0.9575	0.9666

Source : Authors' calculation

These results show that is not easy to discriminate between the exponential model and inverse Gaussian undervariation or inverse Gaussian overvariation alternatives.

4. Concluding remarks

In this paper, we present critical regions based on exponential asymptotic test statistic. The simulations showed good results. The results in Tables 1-3 show that the extreme quantiles of the distribution of $T_{n,E}$ are approximated with sufficient accuracy by those of the centred and reduced gaussian distribution. Furthermore, the results in Tables 4-5 show that it is difficult to discriminate the exponential model with respect to undervariation or overvariation alternatives. However, a natural question arises here: how can the exponential model be distinguished from under/overvariation alternatives ? Answering this question is not an easy task. Furthermore, work is in progress in this direction.

Refereces

Abid, R., Kokonendji, C.C. and Masmoudi, A. (2020). Geometric Tweedie regression models for continuous and semicontinuous data with variation phenomenon. *AStA Advances in Statistical Analysis, 104*, 33-58.

Barlow, R.A., Proschan, F. (1981). Statistical Theory of Reliability and Life Testing : Probability Models, *To begin with*. *Silver Springs*, MD.

Kokonendji, C.C., Touré, A.Y. and Sawadogo, A. (2020). Relative variation indexes for multivariate continuous distributions on $[0, \infty)^k$ and extensions, *AStA Advances in Statistical Analysis*, *104*, 285-307.

Lam, H-K. (1980). Remarks on the distribution of the sample variance in exponential sampling. *Communications in Statistics- Simulation and Computation, 9*, 639-647.

Core Team, $\mathcal{R}$: A Language and Environment for Statistical Computing (2025). *$\mathcal{R}$ Foundation for Statistical Computing*, Vienna.

Touré, A.Y., Dossou-Gbété S. and Kokonendji, C.C. (2020). Asymptotic normality of the test statistics for the unified relative dispersion and relative variation indexes. *Journal of Applied Statistics ,47*, 2479-2491.

Touré, A.Y., Dossou-Gbété, S. and Kokonendji, C.C. (2022). Correction notice to : Asymptotic normality of the test statistics for the unified relative dispersion and relative variation indexes. *Journal of Applied Statistics, 49*, 1612-1614.