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Journal of Applied Economics and Management of Organizations (JAEMO) publie les manuscrits scientifiques dans le domaine des sciences économiques et de gestion ainsi que dans les domaines très proches. De nos jours, les sciences économiques et de gestion ont beaucoup progressé au point qu'elles ont fait des alliances avec les sciences voisines. Cette revue se veut faire la promotion de la recherche scientifique très pointue, originale et innovante.

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## **Asymptotic tests of exponential variation index against alternatives**

### ***Tests asymptotiques de l'indice de variation exponentielle contre des alternatives***

Aboubacar Yacouba TOURE<sup>1</sup> – Amadou BAMBA<sup>2</sup> – Alhousseyni Amadou MAÏGA<sup>3\*</sup>

*1- Université des Sciences Sociales et de Gestion de Bamako, Mali ;*

*[toureboubacaryacoubaaliou@gmail.com](mailto:toureboubacaryacoubaaliou@gmail.com)*

*2- Université des Sciences Sociales et de Gestion de Bamako ; Laboratoire de Recherche en Economie Appliquée au Développement ; [abambisco@gmail.com](mailto:abambisco@gmail.com)*

*3\* - (Corresponding author) ; Université des Sciences Sociales et de Gestion de Bamako ; Ministère de l'Economie et des Finances ; [maigis@yahoo.fr](mailto:maigis@yahoo.fr)*

#### **Abstract**

Variation index with respect to exponential is a classical tool that are widely used in Statistic. This paper aims to gather and study the empirical probabilities of type II error with respect to undervariation and overvariation alternatives. We establish some hypotheses and critical regions and propose numerical studies based on these critical regions. Some concluding remarks are made, including challenging problems.

**Keywords:** Continuous distribution, equivariation, overvariation, probability of type I error, probability of type II error, undervariation.

**Classification JEL :** C15

#### **Résumé**

*L'indice de variation par rapport à la loi exponentielle est un outil classique largement utilisé en statistique. Cet article vise à rassembler et à étudier les probabilités empiriques d'erreur de type II face aux alternatives de sous-variation et de sur-variation. Nous établissons plusieurs hypothèses ainsi que des régions critiques et proposons des études numériques fondées sur ces régions critiques. Quelques remarques conclusives sont formulées, notamment en mettant en évidence des problèmes ouverts et des pistes de recherche.*

**Mots-clés :** Distribution continue, équi-variation, sur-variation, probabilité d'erreur de type I, probabilité d'erreur de type II, sous-variation.

**Classification JEL :** C15

## 1. Introduction

In statistical modelling which is an important step of data analysis in many fields of scientific research, one needs to specify a probability distribution that accounts as accurately as possible the variability observed in the data. In practice it is often considered that certain phenomena such as over/undervariation or zero mass for continuous data (e.g., [Abid et al. 2020](#)).

[Abid et al. \(2020\)](#) have introduced the exponential variation index (VI), so-called Jørgensen variation index, for continuous random variable  $X$  on  $(0, \infty)$  as  $VI(X) = Var X / (\mathbb{E}X)^2$ . It can be viewed as the squared coefficient of variation and, also, it is used in the framework of reliability to discriminate distribution of increasing/decreasing failure rate on the average (IFRA/DFRA) ; see, e.g., [Barlow and Proschan \(1981\)](#) in the sense of the coefficient of variation. A multivariate extension has been well defined and discussed in literature ([Kokonendji et al. 2020](#)). There are many interpretable mechanisms leading to this phenomenon which makes it possible to classify count distributions and makes inference; see, e.g., [Touré et al. \(2020,2022\)](#) for approximative statistical tests. It is not easy to construct an exact statistical test for the index VI based on its plug-in estimator in order to assess the amount of the variability ; see, e.g., [Lam \(1980\)](#) ; [Touré et al. \(2020,2022\)](#).

Furthermore [Touré et al. \(2022\)](#) have introduced the following test statistic :

$$T_{n,V} = \sqrt{\frac{n}{2 + V''(\bar{X}_n)}} \left( \frac{S_n^2}{V(\bar{X}_n)} - 1 \right) \rightsquigarrow \mathcal{N}(0; 1). \quad (1)$$

where  $V''(\cdot)$  denotes the second derivative of the variance function,  $S_n^2$  is the sample variance,  $\bar{X}_n$  is the sample mean,  $\rightsquigarrow$  stands for convergence in distribution and  $\mathcal{N}(0; \tau^2)$  is the centered normal distribution with variance  $\tau^2 = 2 + V''(\mu) > 0$ .

In their work, the authors used the test statistic  $T_{n,V}$  to define critical regions for two-tailed and one-tailed tests. However, they did not attempt to determine the power of these tests under alternatives when the exponential model is inadequate. To our knowledge, no study has established the power of exponential tests for alternatives to the inadequacy of the exponential model.

The two main goals of this paper are, firstly, to propose asymptotic two-tailed and one-tailed exponential tests and, then to propose undervariation and overvariation alternatives in order to study the power of these tests. To get this treatment, we derive from (1) the statistic  $T_{n,E} = \sqrt{\frac{n}{4}} \left( \frac{S_n^2}{\bar{X}_n^2} - 1 \right) \rightsquigarrow \mathcal{N}(0; 1)$  with respect to exponential model and which we will use for the empirical probabilities of type I error in two-tiled and one-tiled tests.

The rest of the paper is structured as follows. Section 2 will be devoted to present the hypotheses to test and critical regions of the test statistics that have produced good results. In Section 3 we analyse the results of some numerical studies. Finally, Section 4 concludes with some remarks and possible extensions.

## 2. Hypotheses to test and critical regions

In this Section, we present the following five families of the hypotheses to test with their corresponding critical regions (1) and for which we find them interesting in practice :

- (i)  $H_{0,EE}: \sigma^2 = \mu^2$  against  $H_{1,EE}: \sigma^2 \neq \mu^2$  for the two-tiled test with respect to the exponential equivariation and, therefore,  $H_{0,EE}$  is rejected if

$$T_{n,EE} = \left| \sqrt{\frac{n}{4}} \left( \frac{S_n^2}{\bar{X}_n^2} - 1 \right) \right| > u_{1-\alpha/2}; \quad (2)$$

- (ii)  $H_{0,EU}: \sigma^2 = \mu^2$  against  $H_{1,EU}: \sigma^2 < \mu^2$  for the one-tiled test with respect to the exponential undervariation and, therefore,  $H_{0,EU}$  is rejected if

$$T_{n,EU} = \sqrt{\frac{n}{2}} \left( \frac{S_n^2}{\bar{X}_n^2} - 1 \right) < u_\alpha; \quad (3)$$

- (iii)  $H_{0,EO}: \sigma^2 = \mu$  against  $H_{1,EO}: \sigma^2 > \mu$  for the one-tiled test with respect to the exponential overvariataion and, therefore,  $H_{0,EO}$  is rejected if

$$T_{n,EO} = \sqrt{\frac{n}{2}} \left( \frac{S_n^2}{\bar{X}_n^2} - 1 \right) > u_{1-\alpha}; \quad (4)$$

- (iv)  $H_{0,IG}: \sigma^2 \geq \mu^3$  against  $H_{1,IG}: \sigma^2 < \mu^3$  for the undervariataion with respect to inverse Gaussian distribution and, thus,  $H_{0,IG}$  is rejected if

$$T_{n,IG} = \sqrt{\frac{n}{2+6\bar{X}_n}} \left( \frac{S_n^2}{\bar{X}_n^3} - 1 \right) < u_\alpha; \quad (5)$$

- (v)  $H_{0,IG}: \sigma^2 \leq \mu^3$  against  $H_{1,IG}: \sigma^2 > \mu^3$  for the overvariataion with respect to inverse Gaussian distribution and  $H_{0,IG}$  is rejected if

$$T_{n,IG} = \sqrt{\frac{n}{2+6\bar{X}_n}} \left( \frac{S_n^2}{\bar{X}_n^3} - 1 \right) > u_{1-\alpha}. \quad (6)$$

These critical regions have led us to carry out simulation studies which results can be found in Tables 1-5 in the next Section.

## 3. Simulation studies

In this Section, we first present the empirical probabilities of type I error for the two-tailed and one-tailed exponential tests. Then, we compute the empirical probabilities of type II error for the inverse Gaussian undervariataion and inverse Gaussian overvariataion alternatives, respectively. All this simulations work was carried out using the  $\mathcal{R}$  software ( $\mathcal{R}$  Core Team, 2025).

The extreme quantiles of the distribution of a test statistic are often those of interest for the implementation of a hypothesis test based on this statistic. Good approximations are therefore needed for these extreme quantiles when the exact distribution of the test statistic is unknown.

The aim of simulations whose results are presented below is to evaluate empirically the quality of the approximation of the extreme quantiles of the test statistic  $T_{n,V}$  by the nominal quantiles corresponding to the limit laws with respect to the sample sizes  $n$ . The simulation study was designed with

- sample sizes  $n = 30, 50, 100, 500$  and  $1000$  ;

- Poisson mean  $\mu = 0.5, 1, 5, 10$  and  $20$  ;
- number of trials of binomial distribution  $N = 25$  and  $100$  ;
- parameter of negative binomial distribution  $\lambda = 25$  and  $100$  ;
- significance level  $\alpha = 0.05$  ;
- number of replications  $r = 50000$ .

Note that we performed 50000 replications for each simulated sample.

### 3.1. Empirical probabilities of type I error

Given that the critical region of the test is determined from the asymptotic distribution of  $T_{n,E}$ , we expect the probabilities of type I error to deviate from the nominal value corresponding to these critical regions. A probability of type I error significantly greater than the nominal value 0.05 would indicate that the precision of the approximation of the extreme quantiles of the distribution of test statistics by those of its limiting distribution is not sufficient. The results of probabilities of type I error for the two-tailed and one-tailed tests are reported in Tables 1-3 where values above 0.05 are in bold.

#### 3.1.1. Exponential two-tiled test

The results in Table 1 show that the empirical probabilities of type I error are lower than the nominal value 0.05 and whatever the sample size  $n$ . Therefore, one can said that the extreme quantiles of the distribution of  $T_{n,E}$  are approximated with sufficient accuracy by those of the centred and reduced gaussian distribution.

**Table 1 : Empirical probabilities of type I error with respect to sample size  $n$  for exponential two-tiled test (i).**

$n$	$\mu = 0.5$	$\mu = 1$	$\mu = 5$	$\mu = 10$	$\mu = 20$
<b>30</b>	0.0315	0.0314	0.0316	0.0323	0.0312
<b>50</b>	0.0353	0.0341	0.0333	0.0344	0.0317
<b>100</b>	0.0395	0.0384	0.0388	0.0385	0.0389
<b>500</b>	0.0462	0.0461	0.0456	0.0453	0.0467
<b>1000</b>	0.0473	0.0485	0.0472	0.0491	0.0468

Source : Authors' calculation

#### 3.1.2. Exponential one-tiled test

For 2-3, the empirical probabilities of type I error are lower than 0.05 or not significantly different from this value and whatever the sample size  $n$ . It is difficult to choose between undervariation and overvariation in view of the results. We cannot discriminate between the equivariation exponential model and undervariation or overvariation.

**Table 2 : Empirical probabilities of type I error with respect to sample size  $n$  for exponential one-tiled undervariation test (ii).**

$n$	$\mu = 0.5$	$\mu = 1$	$\mu = 5$	$\mu = 10$	$\mu = 20$
<b>30</b>	0.0034	0.0039	0.0034	0.0031	0.0033
<b>50</b>	0.0106	0.0095	0.0102	0.0097	0.0101
<b>100</b>	0.0192	0.0193	0.0196	0.0209	0.0195
<b>500</b>	0.0364	0.0368	0.0355	0.0390	0.0379
<b>1000</b>	0.0417	0.0417	0.0405	0.0419	0.0407

Source : Authors' calculation

**Table 3 : Empirical probabilities of type I error with respect to sample size  $n$  for exponential one-tiled overvariation test (iii).**

$n$	$\mu = 0.5$	$\mu = 1$	$\mu = 5$	$\mu = 10$	$\mu = 20$
<b>30</b>	0.0464	0.0476	0.0457	0.0463	0.0460
<b>50</b>	0.0498	<b>0.0501</b>	<b>0.0508</b>	<b>0.0504</b>	<b>0.0503</b>
<b>100</b>	<b>0.0504</b>	0.0477	<b>0.0506</b>	0.0440	0.0465
<b>500</b>	0.0463	0.0448	0.0470	0.0450	0.0447
<b>1000</b>	0.0477	0.0462	0.0466	0.0464	<b>0.0509</b>

Source : Authors' calculation

Thus, the extreme quantiles of the distribution of  $T_{n,E}$  are approximated with sufficient accuracy by those of the centred and reduced gaussian distribution.

### 3.2. Empirical probabilities of type II error

In this Section as an alternative to the inadequacy of the exponential model, we choose the inverse Gaussian distribution for both undervariation and overvariation. The empirical probabilities of type II error are given in Tables 4 and 5.

#### 3.2.1 Inverse Gaussian as under/over-variation alternative

The inverse Gaussian distribution can be used for both undervariation and overvariation alternatives when the results observed in the data do not allow the exponential distribution to be validated. The inverse Gaussian distribution is defined as follow :

$$f(x; \mu, \lambda) = \left(\frac{\lambda}{2\pi x^3}\right)^{1/2} \exp\left\{-\frac{(x-\mu)^2\lambda}{2\mu^2 x}\right\} \mathbf{1}_{(0,\infty)}(x); \quad (7)$$

with  $\mu > 0, \lambda > 0$ . The mean and variance are  $\mu$  and  $\mu^3/\lambda$ , respectively.

With regard to Tables 4-5, the empirical probabilities of type II error are well above 0.05 and whatever the sample size  $n$ .

**Table 4 : Empirical probabilities of type II error with respect to sample size  $n$  for inverse Gaussian undervariation alternative test (iv) when  $\lambda = 3$**

$n$	$\mu = 0.5$	$\mu = 1$	$\mu = 5$	$\mu = 10$	$\mu = 20$
<b>30</b>	0.9989	0.9993	0.9995	0.9997	0.9999
<b>50</b>	0.9943	0.9975	0.9980	0.9988	0.9997
<b>100</b>	0.9858	0.9879	0.9896	0.9923	0.9939
<b>500</b>	0.9867	0.9871	0.9887	0.9913	0.9935
<b>1000</b>	0.9597	0.9668	0.9717	0.9899	0.9918

Source : Authors' calculation

**Table 5 : Empirical probabilities of type II error with respect to sample size  $n$  for inverse Gaussian overvariation alternative test (v) when  $\lambda = 1$**

$n$	$\mu = 0.5$	$\mu = 1$	$\mu = 5$	$\mu = 10$	$\mu = 20$
<b>30</b>	0.9597	0.9719	0.9968	0.9995	0.9999
<b>50</b>	0.9535	0.9632	0.9910	0.9977	0.9996
<b>100</b>	0.9496	0.9555	0.9798	0.9916	0.9981
<b>500</b>	0.9437	0.9465	0.9567	0.9662	0.9771
<b>1000</b>	0.9431	0.9419	0.9521	0.9575	0.9666

Source : Authors' calculation

These results show that is not easy to discriminate between the exponential model and inverse Gaussian undervariation or inverse Gaussian overvariation alternatives.

#### 4. Concluding remarks

In this paper, we present critical regions based on exponential asymptotic test statistic. The simulations showed good results. The results in Tables 1-3 show that the extreme quantiles of the distribution of  $T_{n,E}$  are approximated with sufficient accuracy by those of the centred and reduced gaussian distribution. Furthermore, the results in Tables 4-5 show that it is difficult to discriminate the exponential model with respect to undervariation or overvariation alternatives. However, a natural question arises here: how can the exponential model be distinguished from under/overvariation alternatives ? Answering this question is not an easy task. Furthermore, work is in progress in this direction.

#### Refereces

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